Models, modeling, perturbation problems

What is a model

1. A *model* is the concretization (realization, substantiation) of the way (the filter) we (think to) see reality. The concretization is mathematical, the relation with reality is assumed.
2. A *model* = **[{** a set of smallest elements (entities) **} + {** their properties and interrelations **}]**
3. A *model* should be complete, consistent and without contradictions.
4. “Reality” is by itself also a model: the best model that science at large can provide.
    a. Presently, “reality” is considered to be the Standard Model described by quantum mechanics for the small and the theory of relativity for the large. It is known to be the best we have, even though it is not 100% complete and in some sense not 100% entirely free of contradictions.
    b. Most people have their own model of reality. They ignore elements that they don’t understand or believe, and add features or elements that serve to explain certain personal points of view, but usually have not stand the tests of science. This is true for both the man in the street and the scientist of any discipline. It just goes to show that any model may be inconsistent or contradictory.
5. Popular models of more or less restricted subareas are
    a. Maxwell-Boltzmann model of atoms and molecules
    b. Navier/Stokes/Euler models of media that we experience as continuous, being the continuous limit of “small particles” with point-wise properties and relations.
    c. Newton models of larger objects with simple mechanical properties and relations.
    d. Wave models: when we deal with waves of typical wavelength $\geq\lambda$, geometrical details of interacting objects of size much less than λ can (normally) not be seen by the wave and do not (normally) have to be included in the model.
    e. A plumber’s model of a household piping system: water pipes of a length, large compared to their diameter, connecting knees, and a certain flow resistance depending on pipe length and number of knees.
    f. Sociologists’ models of groups of people with properties and relations.
    g. Psychologists’ models of individuals with properties and relations with external stimuli.
6. Although we aim for models that include the important and ignore the unimportant, this distinction is not always obvious straight away. Therefore, scientifically the most interesting effects are those where our modeler’s intuition fails and a singular coupling exists between a seemingly unimportant detail and an important feature. As long as we cannot solve the “best” model routinely for all our problems, we have to rely on a certain amount of intuition, and to know where intuition fails is just as important as the model itself.
7. At least partially, we can order models from simple to more complex, usually by inclusion of features, properties or relations, or by refining the smallest elements. This is important to realize when we construct a model.
8. Sometimes models are difficult to compare, for example deterministic models and stochastic models. Maybe this is still possible if we consider the level of being stochastic as a property that can be varied from small to large.

What is modeling

1. Modeling is making a judicious selection of the smallest elements and relations that are relevant for our problem, and assemble this into a consistent and contradiction-free model. This selection is based on our intuition, taking into account (1) the questions to be answered, (2) the knowledge of the problem we have, (3) our mathematical skills, (4) the money available (or: the importance of the problem), (5) the time available, (6) consistency, completeness, uniqueness of the solution.

2. Usually, we end up with a hierarchy of models because our intuition is not enough to predict or assess all implications of including or excluding certain elements or relations.

3. By analyzing the models, from simple to complex, we gradually form an idea of the optimal model for the problem at hand. Usually the optimum is the simplest model (smallest number of variables, parameters, and/or assumptions) that can be formulated, following Occam's razor principle.

4. Sometimes the same model can give more information if we are smarter in our analysis; sometimes more information can only be provided by more complex, more comprehensive models.

Perturbation problems

1. A model is usually selected (i) by ordering the potentially relevant effects from important to negligible, and (ii) by identifying a level of complexity of the model beyond which all effects are too unimportant for our use.

2. Hence, by construction, the effects included in the final model will also vary in their level of importance. Some will be large and some will be small (albeit not small enough to ignore).

3. In many cases, this relative smallness can be quantified and, by proper rescaling, made explicit in the form of a problem parameter. As a result, most practical modeling problems present themselves naturally as a perturbation problem, with an associated small parameter.

4. If we are lucky enough, this perturbation problem is amenable to asymptotic analysis, with simpler and more transparent results in the form of functional relations, than is usually provided by a comprehensive approach of the full problem. Invariably, these results allow qualitative, and sometimes even useful quantitative, understanding of the problem.

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