2005-01-2436

A modification of the SEA equations: A proposal of how to model damped car body systems with SEA

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ABSTRACT

The assumption of diffuse (constant) field response limits the applicability of SEA for analysis of structure-borne sound in automotive structures as these are damped and, thus response decay with distance.

A relatively simple modification of the SEA equations can be shown to solve the problem of decay with distance across subsystems. It appears that the modification can be implemented into SEA software without too much problem.

An analytical model is used to demonstrate the errors when the conventional SEA approach is used and the merit of modified SEA equations. A generic firewall, car floor and rear wall is used as showcase.

INTRODUCTION

Statistical Energy Analysis (SEA) has always been limited by the assumptions on weak coupling and reverberant conditions. The assumption on reverberance is needed for the reasons that the modal density should be sufficiently high for a statistical mode count to be accurate and because the response should be uniform across the subsystem, i.e. for subsystem field to be diffuse (without a pronounced direction).

A subsystem with high damping, radiation outdoors, or direct field sound transmission between panels show a response that decays with distance. The response field can not be described by a single response value. Such situations are not handled by conventional SEA.

The SEA equations express power as a function of subsystem energy. Subsystem energy is the integral of the subsystem energy density, which in turn is the local subsystem response times the local density, e.g. surface weight times the square of vibration velocity. In other words, there is not a fundamental limitation in the SEA equations as to why the method should be restricted to non-decaying fields as long as energy flows are accurate.

The SEA equation system needs to be slightly reworked to cope with the fact that response decays across subsystems and new routines must be developed to derive local subsystem response values as these no longer are constant. The latter task is expected to be more laborious than the former.

There are alternative routes to the treatment of decaying fields in predictive SEA and some authors have worked at alleviating SEA from the diffuse field assumption.

Maidanik [1] used room acoustic concepts to develop an extension of SEA theory. Maidanik divides subsystem energy into stored and direct energies where direct energy is associated with the energy that impinges on the first boundary of a subsystem. A wave tracing approach is used to distinguish between stored and direct energies at receiving subsystems.

Heron [2] handles indirect transmission (referred to as tunneling and non-resonant energy transfer) and damping across subsystems in Advanced SEA (ASEA). ASEA divides energy into free- and fixed subsystem energy, and is an iterative solution based on a ray theory approach where results converge with iteration. Heron states that ASEA “can be extended to plate networks although its actual implementation could well be computationally expensive, as compared with standard SEA”.

Finnveden [3] has shown for in-plane wave motion between coupled beams that decay with distance can be incorporated into SEA. Additional degrees of freedom are introduced for the subsystem to handle the decaying field, i.e. the subsystem is segmented into three subparts and the averaged modal energies of these become the new subsystem degrees of freedom. The interrelation between the three subsystem degrees of freedom is a complete 3 by 3 matrix.

The approach that is suggested in this paper is approximate and closer in application to conventional SEA than the other approaches.

THEORY

The use, interpretation and limitations of the power balance is discussed in this section.

CONVENTIONAL SEA POWER BALANCE

Conventional SEA describes the coupling between three subsystems connected in series, Figure 1, as

$$\begin{Bmatrix}\Pi_1\\ \Pi_2\\ \Pi_3\end{Bmatrix}=\omega\begin{bmatrix}\eta_1+\eta_{12} & -\eta_{21} & 0\\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32}\\ 0 & -\eta_{23} & \eta_3+\eta_{32}\end{bmatrix}\begin{Bmatrix}E_1\\ E_2\\ E_3\end{Bmatrix}, \quad (1a)$$

where the angular frequency is ω, the Coupling Loss Factor (CLF) between subsystems j and k is η_{jk}, the Dissipation Loss Factor (DLF), the power input to and response of subsystem j are η_j, Π_j and E_j, respectively [4].

Equation (1a) can be shown to be incomplete. An indirect coupling between subsystems 1 and 3 should be included that relate the non-resonant energy flow across the intermediate subsystem (2), [5]. The importance of the indirect coupling varies with coupling strength between subsystems and subsystem damping. Cases can be identified in which the indirect coupling is dominant as well as where it can be discarded.

It has been proposed, e.g. [6,7], that the indirect coupling can be estimated as

$$\gamma'_{13}\approx\frac{\eta_{12}\eta_{23}}{\eta_2}, \text{ and } \gamma'_{31}\approx\frac{\eta_{32}\eta_{21}}{\eta_2}, \quad (1c,d)$$

where it is inserted into the power balance as

$$\begin{Bmatrix}\Pi_1\\ \Pi_2\\ \Pi_3\end{Bmatrix}=\omega\begin{bmatrix}\eta_1+\eta_{12}+\gamma'_{13} & -\eta_{21} & -\gamma'_{31}\\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32}\\ -\gamma'_{13} & -\eta_{23} & \eta_3+\eta_{32}+\gamma'_{31}\end{bmatrix}\begin{Bmatrix}E_1\\ E_2\\ E_3\end{Bmatrix} \quad (1d)$$

Previous work shows that the indirect transmission data magnitude is quite well approximated by eq (1c,d) and that an inverse analysis scheme that includes indirect couplings can handle cases like the one examined herein, [5]. The use of such coupling data in predictive SEA will therefore be tested as well.

One situation in which the direct- and indirect- couplings are weak was identified by the author. The influence from the indirect coupling can be discarded for this case. Coupling data used in the analysis was predicted with high accuracy. One may then think that SEA should be able to accurately predict response. However, gross errors were found to arise for the predicted subsystem response (3) when exciting in subsystem 1 and vice versa. Satisfactory results were obtained for subsystems 1 and 3 when exciting in subsystem 2. Cleary, this case reveals a situation for which the SEA power balance does not do the job properly.

A physical configuration in which direct couplings are weak and indirect transmission can be discarded is when all three subsystems are highly damped. Another configuration is when only the intermediate subsystem is highly damped. The error in response at the end of the chain relates to the fact that response decays across the intermediate subsystem. The reason why the SEA power balance equation fails in spite of accurate coupling and loss factor data must be attributed to the fact that response decays across the intermediate subsystem.

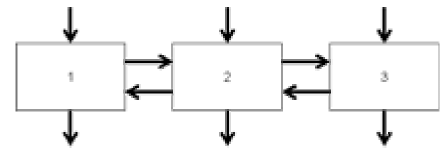


Figure 1. A) Three subsystem division. The logical division between the blocks is at the common plate joints. Arrows aligned inwards a subsystem imply a entry of power whereas arrows aligned outwards a subsystem imply a exit of power.

MODIFIED SEA POWER BALANCE

Before continuing, the reader is reminded that the SEA power balance express input power as a function of subsystem energy. Subsystem energy is the integral of the subsystem energy density. Therefore, the concept of subsystem energy can cover also the case in which the response is not uniform across the subsystem, but the subsystem energy density (or the subsystem response) can not be predicted from subsystem energy alone.

To further strengthen the difference between subsystem energy, subsystem response and the SEA equations, consider the case of two highly damped subsystems, Figure 2. Exciting a subsystem yields a constant energy density across this subsystem. The response of the receiving subsystem shows decay away from the junction. The subsystem energy is accurately predicted by the SEA power balance. However, the subsystem response will be poorly predicted from the subsystem energy.

A decaying subsystem response can not be predicted from subsystem energy alone. The use of the transmitted energy flow is a better choice as the decaying subsystem response can be predicted from this data plus information concerning the subsystem geometry.

The decay with distance, x_j, for the energy density of a freely propagating plane bending wave is

$$e^{\frac{-\pi\eta_j x_j}{2\lambda_j}}, \quad (2)$$

where the loss factor, wavelength and propagation distance of system j are η_j, λ_j. and x_j. , respectively.

Rain-on-the-roof excitation of a system yields a response that truly is uniform across the subsystem even when damping is high. Figure 3(b,c) shows a plot of the energy density distributions when excitation occurs at subsystems 1, 2 and 3, respectively. The energy density at the joint between subsystems 1 and 2 is considerably higher than the energy density at the joint between subsystems 2 and 3 when excitation occurs at subsystem 1. The situation is similar when excitation occurs at subsystem 3. However, the energy density is identical at both ends of subsystem 2 when excitation occurs at subsystem 2.

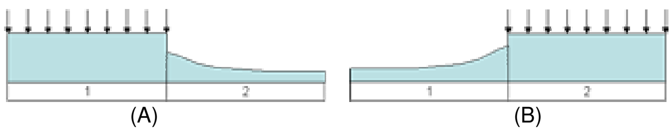


Figure 2. The case of a highly damped 2-subsystem configuration. A) Excitation at subsystem 1. B) Excitation at subsystem 2.

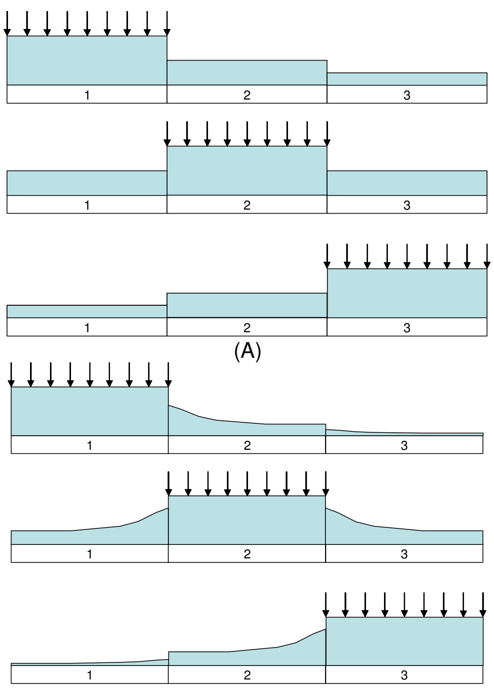


Figure 3. Energy density distribution across subsystems for various cases of rain-on-the-roof excitation. A) Situation as idealized by conventional SEA. B) Expected situation when highly damped.

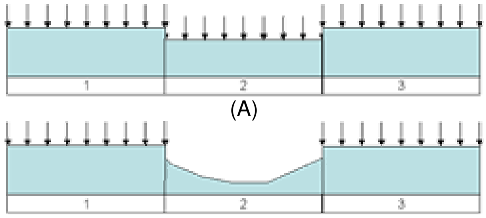


Figure 4. Energy density distribution across systems for various cases of rain-on-the-roof excitation. A) Excited at all subsystems. B) Excited at subsystems 1 and 3.

These observations imply that the SEA power balance can do the job at correctly predicting the energy flows into subsystems 1 and 3 when excitation occurs at subsystem 2. Similarly, the SEA power balance will correctly predict the energy flows when subsystems 1 and 3 are exposed to excitation. However, the power balance can not handle the cases for which excitation occurs either at subsystem 1 or at subsystem 3 as there is a decay across the intermediate subsystem.

The energy flow across a joint is proportional to the difference in energy density at both sides of the joint. It is the difference in local energy density that decides the transmission, not the total or average energy density of the subsystem. Therefore, a marked drop in energy density across the intermediate subsystem affects the power transmission at the next joint.

Note that the drop in energy density in subsystems 1 and 3 when excitation occurs in subsystem 2 does not pose a problem for the SEA power balance as the energy density is constant across subsystem 2. For similar reasons, the SEA power balance will accurately predict the subsystem energy when subsystems 1 and 3 are simultaneously excited, Figure 4. The response within the intermediate subsystem may be poorly predicted, but the subsystem energy will be accurately predicted by conventional SEA. Also, the case at which all subsystems are excited will be correctly handled by the SEA power balance.

Thus, the SEA power balance needs correction only when excitation occurs at either subsystem 1 or subsystem 3. The decay with distance from joint 1-2 to joint 2-3 is the same when energy flows in the other direction. Therefore, we may get away with the addition of a single correction of the SEA power balance.

The SEA power balance correction should be turned on when excitation occurs either at subsystem 1 or at subsystem 3 and should otherwise be turned off.

A problem in the use of eq (2) is that the decay distance must relate to the average subsystem energy when the decay across the subsystem should be predicted. The position for the average subsystem energy must therefore be determined before the decay factor can be applied.

A weak decay with distance would lead to a position for the average subsystem energy at about half the subsystem length. This is the situation to be expected at low frequency. A strong decay with distance suggests that the average subsystem energy would be positioned in close proximity to a joint of a sender subsystem. This is the expected situation at high frequency.

The position of average subsystem energy can be estimated from eq. (2) as

$$F_j = 1 - \frac{\int_0^{L_j} x e^{ax} dx}{L_j \int_0^{L_j} e^{ax} dx} = \frac{e^{a_j L_j} - a_j L_j - 1}{a_j L_j \left(e^{a_j L_j} - 1\right)}, \qquad (3a)$$

where F_j is the fraction of the subsystem length from the receiving junction at which the average is found, and where

$$a_j = \frac{\pi \eta_j}{2\lambda_j}. \qquad (3b)$$

Note that the position for average subsystem energy changes with frequency, Figure 5. Note also that eq (3a) is approximate as energy density is assumed to drop to zero at L_j. Eq (3a) will err when the portion of decaying subsystem energy is about as large as the reverberant energy portion of a subsystem, i.e. when decay is weak.

The estimate of eq (3a) can probably be improved by estimation of stored- and decaying energy, e.g. as a post-analysis operation and a weighted average between 0.5 for the former and eq (3a) for the latter part. The approach would then become iterative (not tested herein). One may also use an analytical estimation of stored and decaying energy in a manner similar as to when decay radius is calculated in room acoustics .

A bending wave decay factor that caters for the fact that the subsystem energy decays with distance can now be set up as

$$\Delta_j = e^{-a_j L_j F_j \delta_{jkl}}, \qquad (3c)$$

where L_j is the subsystem j length and an extra factor, δ_{jkl} , is added. The factor δ_{jkl} is zero when excitation occurs at either of the connected subsystems k and l and is otherwise unity.

In passing, it can be noted that in-plane wave decay is handled simply by dividing eq (3b) by a factor of two and that it may be possible to handle geometric decay with a procedure similar to the one applied herein.

A straightforward way to compensate for decay across a subsystem is to multiply the coupling data that govern the response in subsystems 1 and 3 with the decay factor of equation (2). The SEA coupling procedure of equation (1) is then modified to

$$\begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21}\Delta_2 & 0 \\ -\eta_{12} & \eta_2 + \eta_{21} + \eta_{23} & -\eta_{32} \\ 0 & -\eta_{23}\Delta_2 & \eta_3 + \eta_{32} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}, \quad (4)$$

where it is noted that the decay factor, Δ_2, would be close to unity when the response field in subsystem 2 is reverberant and that the decay factor is unity when the factor δ_{jkl} is zero.

The reasoning behind the introduction of the decay factor is that the energy density in subsystem 2 will drop across subsystem 2 when excitation occur either in subsystem1 or when it occurs in subsystem 3. For the former situation, the energy flow across joint 1-2 is correctly predicted, while the energy flow across the joint 2-3 is grossly overestimated when decay is not taken into account. Also the re-radiation from subsystem 3 across subsystem 2 will be grossly overestimated when decay is neglected. Therefore, the decay factor must be introduced only at the matrix positions [1,2] and [3,2] as these coupling data terms cause the energy flow overestimation. The decay factor provides a simple scaling factor for the drop in energy density across the subsystem.

Summing the rows, the dissipated power becomes

$$\Pi_1 + \Pi_2 + \Pi_3 = \omega[\eta_1 \ (\eta_2 + (\eta_{21} + \eta_{23})(1 - \Delta_2)) \eta_3] \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}, \quad (5)$$

which shows that the decay factor does add damping to subsystem 2. Therefore, this procedure is approximate for the prediction of subsystem energy.

The damping addition from the decay factor is an artifact of the procedure and leads to an underestimation of the total energy for subsystem 2 when excitation occurs at subsystem 1 or at subsystem 3. However, the underestimation should be slight as the CLF magnitude tends to be significantly smaller than the DLF magnitude when decay with distance matters and because the total energy for subsystem 2 is of little use to us anyhow. The response across subsystem 2 must in any case be post processed with other means to show the spatial response distribution. More importantly, errors are not introduced to the energy flow calculation which is the primary analysis task for the modified SEA power balance of eq (4).

Removing the dissipated power from the input power shows the energy flows

$$\begin{Bmatrix} \Pi_{1->} \\ \Pi_{2->} \\ \Pi_{3->} \end{Bmatrix} = \omega \begin{bmatrix} \eta_{12} & -\eta_{21}\Delta_2 & 0 \\ -\eta_{12} & \eta_{21} + \eta_{23} & -\eta_{32} \\ 0 & -\eta_{23}\Delta_2 & \eta_{32} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (6)$$

where the net energy flow out of subsystem j, is denoted as $\Pi_{j->}$ and we may or may not choose to use the corrected intermediate subsystem energy.

To exemplify, the response at a particular position in subsystem 3 can be computed as

$$v_3^2(x_3) = \frac{\Pi_{3->}}{\omega \eta_3 M_3} e^{\frac{\pi \eta_3 x_3}{2\kappa_3}} \qquad (7)$$

when exciting in subsystems 1 and/or 2 and, where M_3 is the mass of subsystem 3.

Note that this simple response estimation can be used as the energy density at the junction is constant. This may or may not be a good approximation, in case the approach shown herein is extended to cover also geometric decay or in case point connections or short line junctions are analyzed. The response prediction of the local subsystem response must then be refined.

As was above mentioned, Finnvedens work on how to handle subsystems with a decaying field shows that an approach in which decay is handled by division of a subsystem into a chain of smaller segments implies that a complete (all matrix elements) 3 by 3 matrix must be used. This suggests that the commonly employed attempts at a 'SEA theory bug fix' in which subsystems simply are hacked up into several smaller segments will meet with limited success.

Figure 8 shows a comparison where a predictive (AutoSEA) SEA model using a chain of subsystems is used to predict response. The intermediate (floor) subsystem is modeled as a single subsystem, as 4, and as 8 subsystems connected in a chain. The energy for subsystem 3 is computed (we need not divide this subsystem into segments as long as energy is monitored) for the case of excitation in subsystem 1.

The multi-subsystem results of Figure 8 clearly show that the segmentation approach can not be based on first principles, i.e. subsystem geometry, conventional assumptions on coupling data and damping information. The results for the case in which the intermediate subsystem is modeled by a single subsystem is in accordance with the results for eq (1a) in Figure 7(a), which implies that there is nothing wrong with the data the predictive SEA model uses.

The modified SEA coupling approach that was put forward here is motivated mainly by the ease with which it can be implemented into existing analytical SEA codes.

GENERIC FIREWALL, FLOOR AND REAR WALL EXAMPLE

An analytic model of three simply supported plates connected in U-shape is used as the reference model, [5]. A second analytic model for two simply supported plates connected in L-shape, [8], is used to generate coupling data for the subsystems pairs firewall-floor and floor-rear wall, respectively.

The case of excitation in subsystem 1 is the only case shown as excitation in subsystem 3 yields results of a similar quality. The case of excitation in subsystem 2 yield results for all subsystems that are comparable in quality with those of Figure 7(b,c) and is therefore excluded as well.

WORK PROCEDURE

Calculation of coupling data for two L-plate configurations (firewall-floor and floor-rear wall, respectively) generate coupling data for eq (1a-d), and eq (4). The loss factors for subsystems 1 to 3 are known and need not be computed as the proposed procedure is intended for predictive SEA. The decay factor is computed using eq (3c)

There is no theoretical limitation as to why the power balance should not be analyzed in narrow frequency bands. The use of modal counts can be replaced by the use of modal density in predictive SEA. In fact, results improve when analysis is made using narrow frequency bands as we need no longer rely on the assumption that

$$\int A(f)\cdot B(f)df \approx \int A(f)df \cdot \int B(f)df \quad , (8)$$

when computing subsystem energy from the power balance.

The use of wide frequency bands is inspired by the use of third octave- and octave- band analyzers in test work and may have been prompted by computer restrictions of the past. However, it can be alleviated with current computer resources.

All data analysis herein is therefore made using narrow bands, i.e. L-plate coupling data is computed using narrow bands and U-plate response is estimated solving the power balance in narrow frequency bands. Calculations made using AutoSEA are made with a frequency band of 1 Hz as well.

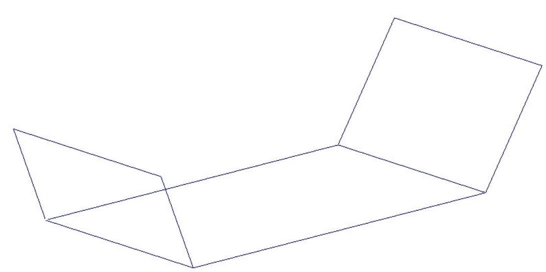
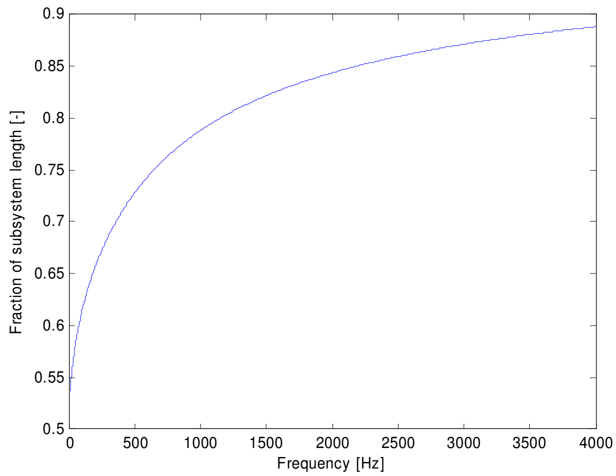
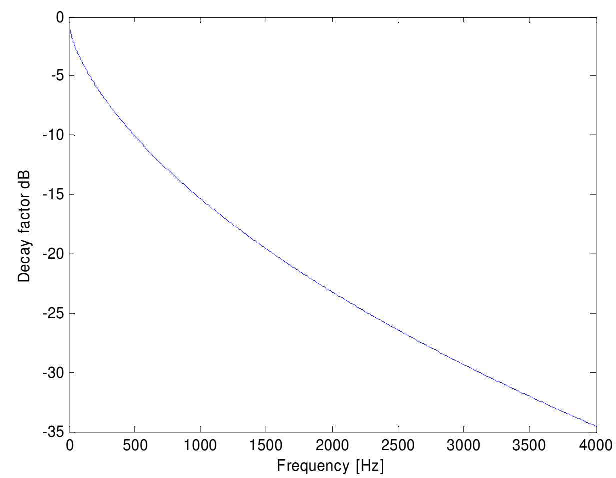


Figure 5. Generic model of firewall, floor and rear panel. The firewall is is the LHS subsystem (1), the floor is the intermediate subystem (2) and the rear wall is the RHS subsystem (3). The width is 1.6 m of all plates, the firewall is 0.6 long, the floor is 2.0 m long, and the rear plate is 0.8 long. All plates are 1 mm thick, are made from steel an have a dissipation loss factor of 10%. The analytical U- and L-plate models incorporate bending only and use simply supported boundary conditions.

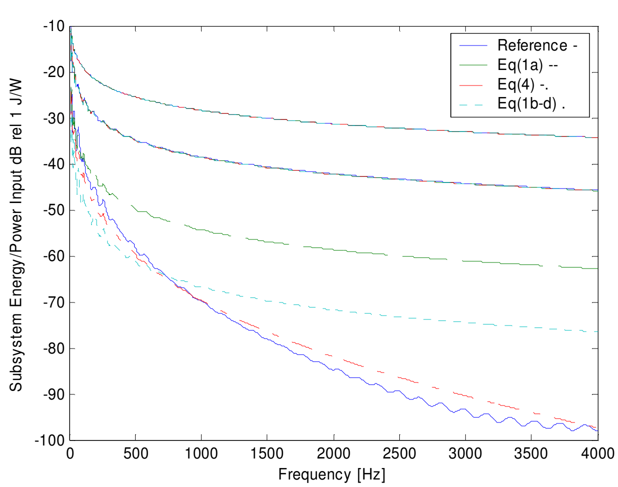


(A)

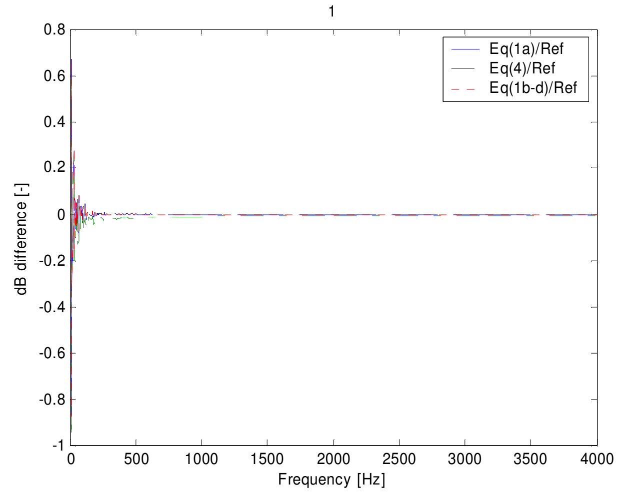


(B)

Figure 6. Data for the floor subsystem. A) Estimated position for the average subsystem energy as a function of frequency for subsystem 2. Note that the plot shows eq (3a), i.e. the fraction of the subsystem length over which decay with distance is expected. B) The decay factor of eq (3c) as a function of frequency.



(A)



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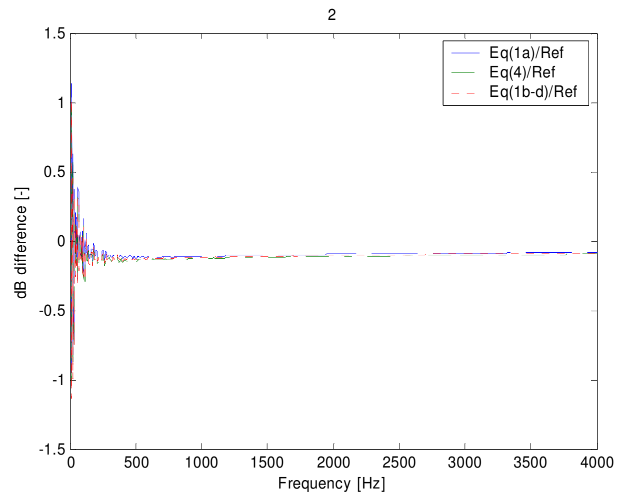
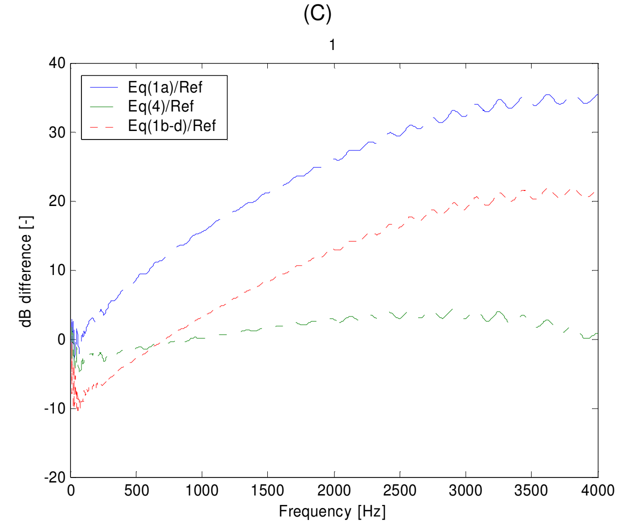


(C)



(D)

Figure 7. Results when exciting at the firewall, subsystem 1.
A) Subsystem responses. **Note** that the subsystem energy estimated using eq (1b-d) is negative. The absolute value is therefore plotted.
B) The difference in computed energy at subsystem 1. C) The difference in computed energy at subsystem 2. D) The difference in computed energy at subsystem 3. The error when using eq (4) varies between -5 dB and +4 dB for the investigated case.

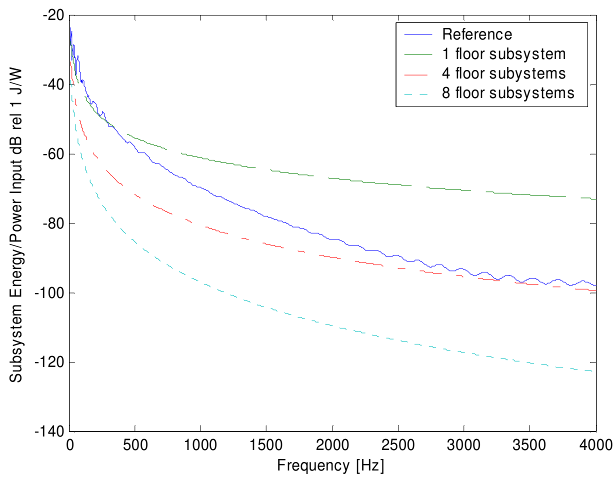


Figure 8. Response in subsystem3 when exciting at subsystem 1. Test of the approach in which decay is approached using a chain of SEA subsystems. The floor subsystem (2) is handled as a single, four or eight subsystems. Transmission across joints is made using moments only. Transmission across the floor multi-subsystem models is made using transverse force and moments. Analysis was made using AutoSEA V 1.5.

CONCLUSIONS

The conventional SEA power balance, as expected, fails to accurately predict the subsystem response when there is significant decay across a subsystem. However, the SEA power balance can still do the job at predicting the subsystem energy and the energy flow when there is decay within a subsystem as long as the energy density at the joint can be foreseen. Put in other words two highly damped subsystems can be handled with the conventional SEA power balance, while transmission across a highly damped subsystem is not accurately handled by the SEA power balance.

It was found that the use of subsystem energy as the primary analysis result must be discarded when dealing with subsystems with a decaying energy density. A better approach is instead to compute energy flow and to derive the subsystem response distribution from the power inputs at its interfaces.

A modification to the SEA power balance in which a decay factor that accounts for the drop in energy density across the transmitting subsystem was introduced and shown to greatly improve the prediction of energy flow and subsystem energy. The suggested procedure is not exact, but improves results as long as the loss factor of the transmitting subsystem is significantly larger than its coupling data, which is expected to be the case when decay matters. The decay factor is triggered such that it is active only when excitation is applied at either of the outer subsystems, i.e. the use of the decay factor is not only a function of the subsystem but depends also on where excitation occurs.

An analytic model of a U-shaped plate that is exposed to rain-on-the-roof excitation was used as reference solution, while coupling data was calculated using an analytic L-plate for two L-plate combinations. Coupling- and loss factor data was combined to compute subsystem response. The example of a generic firewall-floor-rear plate system showed that transmission across the floor can be off by ~35 dB when the conventional SEA power balance is used, while the modified SEA power balance provided results that were within -5 dB and +4 dB of the reference solution for the investigated case.

A comparison between the reference model and the commonly used approach in which subsystems with decay is divided into several segments showed that results greatly differ with the number of segments that are used to approximate the decay. Dividing the floor into four segments captures the response at high frequency but underestimates the response at low frequency. The use of eight subsystems did not provide convergence on the 4 segment model. Therefore, this modeling technique can not be stated to be based on first principles, i.e. data like subsystem size, damping and joint geometry.

A decay factor can easily be introduced into conventional SEA software, while the prediction of subsystem response distribution is more laborious to implement.

The use of a decay factor in the modified SEA power balance is expected to be applicable also for other types of decaying fields, e.g. geometric decay with distance for direct fields.

The investigated example, a generic firewall-floor-rear wall automotive structure is a case with a high degree of decay with distance. Situations in which the decay with distance is influential but less pronounced may benefit from improved estimation of the position for the average subsystem energy. Suggestions for such improvements was made in the discussion but is not attempted herein. So, the reader is cautioned to treat the use of eq (3a) as a first stab at analysis of such situations.

ACKNOWLEDGMENTS

The work reported herein was made at Ingemansson, and is a spin off from research that was made at the Department of Technical Acoustics at Chalmers. The research work was financed by the Swedish Board for Technological Development (NUTEK).

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